Ch. 16: Random Variables

* A numeric value based on the outcome of an event

* We use capital letters to denote random variables
  (like X, Y, Z)

* Example: tossing a coin 3 times and recording the number of heads: X = 0, 1, 2, 3 Heads ~~2.5~~

* Example: # of children in a family: X = 0, 1, 2, 3, ...

Random Variables: 2 types

	* Discrete R.V.	Continuous R.V.
What is it?	* Can list all outcomes	* Outcomes are in an interval * cannot list all
Examples:	* # of heads in 3 coin tosses * # of children in a family	(0, max) * # of hours a light bulb lasts * Time to run a mile
	often whole # outcomes	not whole #'s

<table>
<tr><th></th><th>DISCRETE</th><th>CONTINUOUS</th></tr>
<tr><td>Distribution/ Function</td><td>* Probability Model
<table><tr><th>X</th><th>1</th><th>2</th><th>3</th><th>4</th><th>5</th></tr><tr><td>P(X)</td><td>0.1</td><td>0.3</td><td>0.5</td><td>0.03</td><td>0.07</td></tr></table>* Probability histogram</td><td>* Continuous Distrib.

* Ex: normal distrib., uniform distrib.</td></tr>
<tr><td>Properties of Distrib.</td><td>* Sum up to 1
$P(X \geq 3) = 0.6$
* All disjoint prob. Add prob. to get sums $P(X > 3) = 0.1$</td><td>* Area under curve = 1

* No individual probabilities!! $P(X = 2) = 0$</td></tr>
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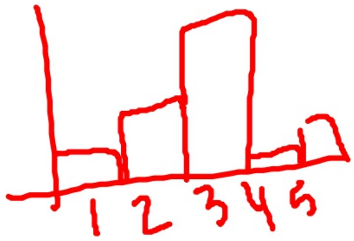
We will focus on DISCRETE Random Variables in this chapter



ample:

1) Let's play a game! You pay \$5 to play. In the game, you get to draw one card from the deck.
    - If you draw the ace of hearts, you win \$100
    - If you draw any other ace, you win \$10
    - If you draw any other heart, you win \$5
    - Any other card, you win nothing
    - We are interested in the amount that we $\boxed{\text{GAIN}}$

$\mu_x = \$-1.35$

$\sigma_x = \$13.80$

a) Create the probability distribution below:
   X = values of the variable. The outcomes
   P(X) = the probabilities of the values. Must sum to 1.

X	\$95	\$5	\$0	-\$5	
P(X)	$1/52$	$3/52$	$12/52$	$36/52$	$= 1 = 52/52$

b) What is the chance that you go home with at least some money? $P(X>0) =$

$P(X \geq 5) =$ $4/52$

c) What is the chance that you ~~win~~ GAIN at least \$10? $P(X \geq 10) = 1/52$

d) What is the probability of not gaining any money?

$P(X \leq 0) = 48/52$

Mean/Std. Deviation

MEAN = Expected Value

* Notation: μ_X μ_H

$E(X)$ $E(H)$

Expected Value = the value we would expect to get if we played the game many, many, many times. A theoretical, long-run average.

weighted avg.

$$\mu_X = \$10.25$$

Formula (how to find it):

- X is a Discrete R.V. with the following distr.:

X	X1	X2	X3 ...	Xn
P(X)	P(X1)	P(X2)	P(X3) ...	P(Xn)

$$X_1 \times P(X_1) + X_2 \times P(X_2) + X_3 \times P(X_3) + \dots + X_n \times P(X_n) =$$

- Find the mean (expected value) by doing the following:

$$\sum X_i \cdot P(X_i)$$

HW	15%
CW	15%
+ Test	70%
	Grade

CALCULATOR:

- L1 = values of variable (top row of table) X
- L2 = Probabilities of each value (bottom row of table) $P(X)$
- STAT → CALC → 1-Var-Stats L1, L2
- Mean = $\overline{x}$ Std. Deviation = σ $\cancel{S}$

μ_X

$E(X)$

σ_X

σ_H

ample:

X	0	1	2	3	4	5
(X)	0.05	0.12	0.18	0.2	0.4	0.05

L_1

L_2

hat is the mean (expected value) and std. deviation?

$\mu_x = 2.93$ $\sigma_x = 1.3058$

ample: Find the expected value (mean) and the std. deviation of the first game above.

1 var stats L_1, L_2

$\bar{x} =$

$\sigma =$

$s =$ ______

mple: New game! We are rolling a die. We are again interested in the amount ~~WIN~~. It costs $5 to play

GAIN

- **Roll a 6, you win $10**
- **Roll a 5, you win $7**
- **Roll a 3 or 4, you win $5**
- **Roll a 1 or 2, you win nothing**

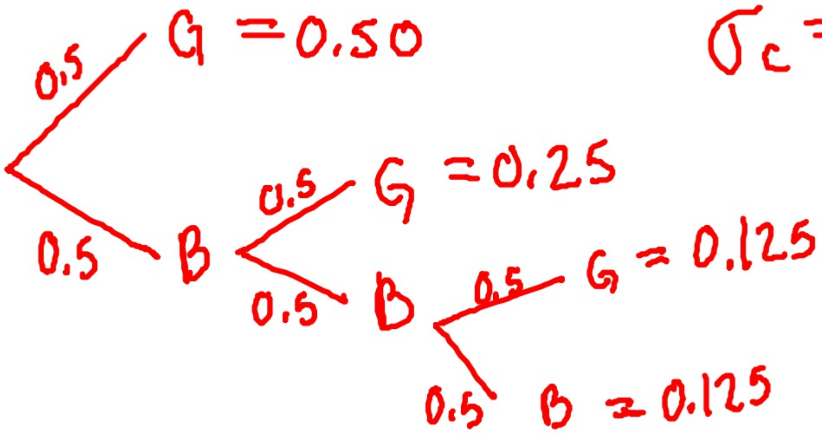
ate the prob. distribution and find the expected value (mean) and standard deviation of the game:

Book examples: p. 383 #5, 8, 23, 7

#children	1	2	3
$P(C)$	0.50	0.25	0.25

$\mu_C =$

$\sigma_C =$



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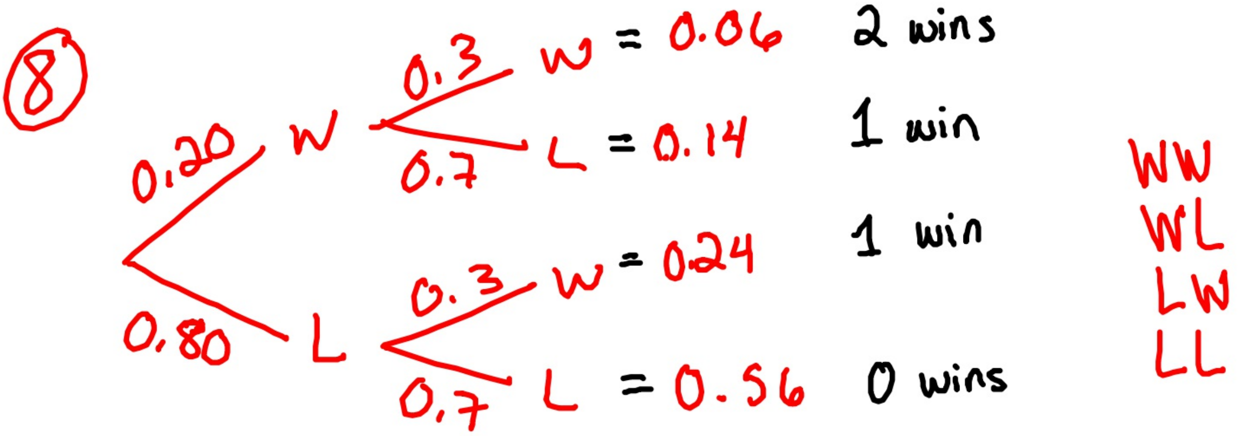
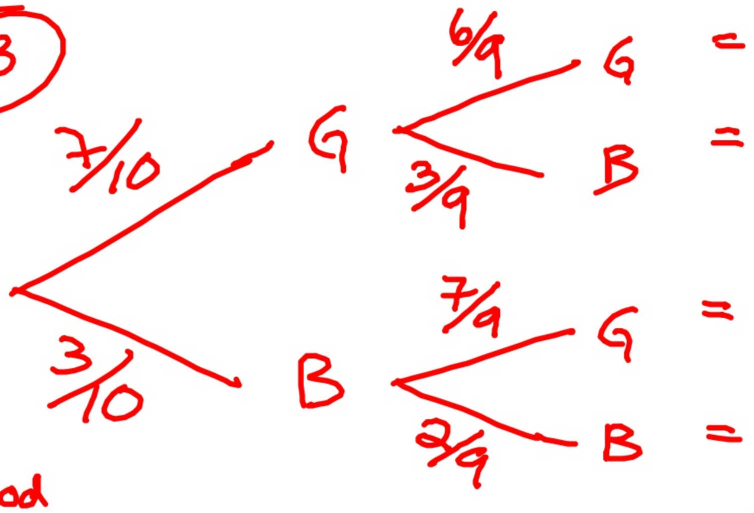


X	\$80,000	\$30,000	-\$10,000	L_1
P(X)	0.06	0.38	0.56	L_2

$E(X) = \mu_X = \$10{,}600$

23



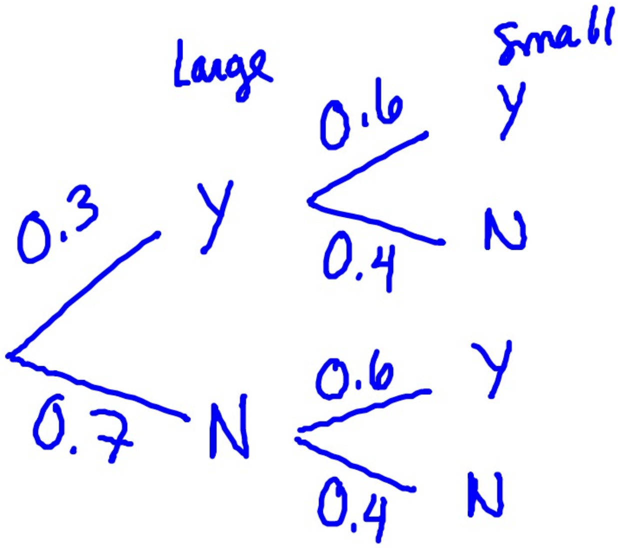
Good

X	0	1	2
P(X)	$\frac{6}{90}$	$\left(\frac{21}{90}\right)+\left(\frac{21}{90}\right)$ $42/90$	$42/90$

E(X) = 1.4

SD(X) = 0.611

7



= YY = 0.18 70,000

= YN = 0.12 50,000

= NY = 0.42 20,000

= NN = 0.28 0

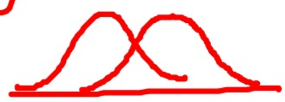
X = Profit	\$0	20,000	50,000	70,000
P(x)	0.28	0.42	0.12	0.18

nsforming Random Variables

- We may want to alter a random variable (add, subtract, multiply, divide the variable)
- We may want to combine it with another random variable

(and Y be independent random variables *Let "a" and "b" be fixed numbers*

$\mu_X = 10$



LES

ltiplication and addition of a variable:

MEAN: $\mu_{a+bX} = a + b \cdot \mu_X$

VARIANCE: $\sigma^2_{a+bX} = b^2 \sigma_X^2$ $\sigma_{a+bX} = b \cdot \sigma_X$

$\sqrt{b^2\sigma_X^2}$

$\boxed{b\sigma_X}$

REMINDER:

Std. Deviation = $\sqrt{\text{var}\,iance}$

$\sigma = \sqrt{\sigma^2}$

REMINDER:

Std. Deviation = $\sqrt{variance}$

$\sigma = \sqrt{\sigma^2}$

(X and Y)

Combining 2 or more variables together:

MEAN: $\mu_{x+y} = \mu_x + \mu_y$

$\sigma_x = 3 \quad 9$

$\sigma_y = 4 \quad 16$

$\mu_{x-y} = \mu_x - \mu_y$

VARIANCE:

$\boxed{\sigma_{x+y}} \longrightarrow \sigma^2_{x+y} = \sqrt{\sigma_x^2 + \sigma_y^2}$

$\neq \sigma_x + \sigma_y$

$\updownarrow$

σ_{x-y}

$\mu_X = 6.2 \quad \sigma_X = 3.1$

EXAMPLES:

1) Random variable X has a mean of 6.2 and a standard deviation of 3.1.
   a. Find the new mean and standard deviation if we multiply by 3

      $\mu_{3X} = 3 \cdot 6.2 = 18.6$

      $\sigma_{3X} = 3 \cdot 3.1 = 9.3$

   b. Find the new mean and standard deviation if we subtract 10

      $\mu_{X-10} = 6.2 - 10 = -3.8$

      $\sigma_{X-10} = 3.1$

   c. Find the new mean and standard deviation if we multiply by 5 and add 10.

      $\mu_{5X+10} = 41$

      $\sigma_{5X+10} = \sigma_{5X} = 15.5$

2) Random variable Y has a mean of 3.4 and a standard deviation of 1.4.

a. Find the mean and standard deviation of X + Y

$\mu_X = 6.2$

$\sigma_X = 3.1$

$\mu_Y = 3.4$

$\sigma_Y = 1.4$

$\mu_{x+y} = 9.6$

$\sigma_{x+y} \Rightarrow \sigma^2_{x+y} = (3.1)^2 + (1.4)^2 = \sqrt{11.57} = 3.401$

b. Find the mean and standard deviation of X − Y

$\mu_{x-y} = 6.2 - 3.4 = 2.8$

$\sigma_{x-y} = 3.401$

c. Find the mean and standard deviation of 2X + 3Y

$\mu_{2x+3y} = 2(6.2) + 3(3.4) = 22.6$

$\sigma_{2x+3y} = (2 \cdot 3.1)^2 + (3 \cdot 1.4)^2 = \sqrt{56.08} = 7.489$

d. Find the mean and standard deviation of 3X + Y − 4

$\sigma_{3x+y} = \sqrt{(3 \cdot 3.1)^2 + 1.4^2}$

) New game! A single dice is rolled and the following occurs:

- Roll a 6, get 40 points
- Roll a 4 or 5, get 10 points
- Roll a 1, 2, or 3, get 0 points

. Write the probability distribution

pts.	40	10	0
P(p)	1/6	2/6	3/6

. Find the mean and standard deviation of the game

$\mu_p = 10$ pts. $\sigma_p = 14.142$ pts.

. Suppose the points are doubled. Find the new mean and standard deviation

$\mu_{2p} = 20$ pts. $\sigma_{2p} = 28.284$ pts.

. Supposed the game is played twice (independently). What is the mean and standard deviation of this?

$\mu_{p+p} = 20$ pts.

$\sigma_{p+p} = \sqrt{14.142^2 + 14.142^2} = 20$ pts.

More examples:

1. Suppose that X is a random variable with $\mu_X = 10$ and $\sigma_X = 2$. Find the following:

   a. μ_{5X} b. μ_{X-3} c. μ_{5X-3}

   d. σ_{5X} e. σ_{X-3}

2. Suppose that X is the random variable from above, and that Y is independent from X with $\mu_Y = 15$ and $\sigma_Y = 3$. Find the following:

   a. μ_{X-Y} b. μ_{X-3Y} c. σ_{X+Y}

   d. σ_{X-Y} e. σ_{5X-2Y}

3. Suppose the mean SAT verbal score is 425 with standard deviation 100, while the mean SAT math score is 475 with standard deviation 100. What can be said about the mean and standard deviation of the combined math and verbal score?

Book examples: p. 384 $\boxed{\#16 \ \& \ 32,}$ ~~24~~, ~~25~~, 38, 40

16) (a) $\mu_X = 2.25$ red lights

(b) $\sigma_X = 1.26$ red lights

$\mu_{X+X+X+X+X} = 11.25$ red lights

$\sigma_{X+X+X+X+X} = \sqrt{5(1.26)^2}$

32) $\mu_{X+X+X+X+X} = 11.25$ red lights $\quad \sigma_{X+X+X+X+X} = 2.82$ red lights

38 $\mu_D = \$100$

$\sigma_D = \$30$

$\mu_C = \$120$

$\sigma_C = \$35$

a) $\mu_{D-C} = -\$20$

b) $\sigma_{D-C} = \sqrt{30^2 + 35^2}$

$= \$46.10$

c) Normal $N(-20, 46.10)$

$$P(\underset{-C}{D} > \underset{-C}{C})$$

$$P(D - C > 0) = \text{normalcdf}(0, E99, -20, 46.10)$$

25) (a) $\mu = 30$, $\sigma = 6$
(b) $\mu = 26$, $\sigma = 5$
(c) $\mu = 30$, $\sigma = 5.39$
(d) $\mu = -10$, $\sigma = 5.39$

(e) $\mu = 20$ $\sigma = 2.83$

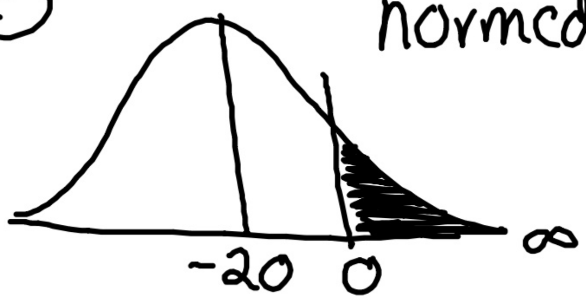
(38) $\mu_D = \$100$ $\mu_C = \$120$

$\sigma_D = \$30$ $\sigma_C = \$35$

(a) $\mu_{D-C} = -\$20$

(b) $\sigma_{D-C} = 46.098 \rightarrow \$46.10$

(c)



normcdf(0, E99, -20, 46.10)

33.22%

40 a) $D + D + C$ = total cost

b) $\mu_{D+D+C} = 100 + 100 + 120 = \$320$

$\sigma_{D+D+C} = \sqrt{30^2 + 30^2 + 35^2} = \$55$

$N(320, 55)$

c) $P(\text{total} > 400) =$

= normcdf(400, E99, 320, 55)

= 0.0729